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Asymptotic estimates of cellular circuit area in arbitrary basis and universal synthesis method[1]

V. S. Zizov (Moscow, Russia)
vzs815@gmail.com

The arbitrary complete bases of cellular circuits of functional and switching elements are discussed in the paper. Previously obtained results of upper and lower estimates are generalized to the arbitrary basis. Lower estimates are proven on power reasons. Upper estimates are constructively built on the ground of universal synthesis method and generalized to the arbitrary basis case.

Keywords: celullar circuit, planar circuit, Shannon function, asymptotic estimates, circuit area, lower estimates, upper estimates.

Универсальный метод синтеза и асимптотические оценки сложности клеточных схем в произвольном полном базисе[1]

В. С. Зизов (Москва, Россия)
vzs815@gmail.com

В настоящей работе рассматриваются произвольные базисы клеточных схем из функциональных и коммутационных элементов. Полученные раннее результаты верхних и нижних оценок обобщаются на произвольный базис. Нижние оценки доказываются исходя из мощностных соображений. Верхние оценки строятся конструктивно на основе универсального метода синтеза для одного базиса, и обобщаются на случай произвольного базиса.

Ключевые слова: клеточные схемы, плоские схемы, функция Шеннона, асимптотические оценки, площадь схемы, нижние оценки.

Introduction

The research of the complexity of descrete functions (the quantity of functional elements and contacts in different models) took place in many previous papers. The curcuit area becomes an important parameter while implementing of the real circuits in area, taking into account their sizes and geometric features, especially in the case of integrated circuits. In the paper is discussed the descrete function computation model similar to circuits of functional elements.

Similar mathematical model was described by Thompson in the monograph [1]. It is the ground for the integrated circuits research, but it

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doesn't take into account the delays during signal propagation in conductors. It is considered to be one of it's diadvantages. Chazelle and Louis [2] proposed a model describing the delays occuring in the circuit.

In 1967 Kravtsov in [3] was chronologically the first to propose similar planar circuit model, consisting of functional and switching elements. He gave the definition of «standard» basis. The model important feature is the Shannon function behaviour i.e. the complexity of the most complex function from n boolean variables. The asymptotics of Shannon function for the cellular circuit model [3] was proved by Albrecht in [4]. The paper showed that the asymptotics of Shannon function looks like $\sigma 2^n$ where $\sigma = const$. The exact constant σ value is still unknown, but from [3] and [4] can be seen that it is situated in the segment $[\frac{1}{4}, \frac{9}{2}]$.

Zizov and Lozhkin for the first time received asimptotically tight bounds $n2^{n-1}$ for the decoder area of order n [5] and lookup function of order n [6]. The lower estimate of Shannon function was improved in [7].

In that paper is discussed the basis from the paper [5] and received upper and lower estimates results are being generalized to the arbitrary basis.

Let's define the function f complexity $A(f)$ as the least of the complexities of cellular circuit Σ implementing f. Let's introduce the Shannon function $A(n)$:

$$A(n) = \max_{f \in P_2(n)} A(f).$$

Here $P_2(n)$ is a set of all boolean functions from n variables.

Theorem 1 (universal synthesis method). *For every boolean function f from n variables there is a cellular circuit S implementing f and repeating it's inputs one-time and has the area estimate:*

$$A(S) \leqslant 2^n + O(n2^{\frac{n}{2}}), \quad n \to \infty.$$

The proof is done by the construction of such a circuit according to the following principle (Pic. 1):

a) let the variables set $x_1, \ldots, x_n$ be divided into two equal parts $x_1, \ldots, x_{n/2}, x'_1, \ldots, x'_{n/2}$;

b) top and left sides are consisting of decoders, constructed according [5] as a conjunctive decoder from the variables $x_1, \ldots, x_{n/2}$ and a disjunctive decoder from the variables $x'_1, \ldots, x'_{n/2}$;

c) intersection lines i, j contain matrix elements $M_{i,j}$ such that they are a functional element & when on that set $x_1, \ldots, x_{n/2}, x'_1, \ldots, x'_{n/2}$ the function takes the value 0 and a switching element in another case.

According to construction every vertical line implements partial function on the fixed set of variables $x_1, \ldots, x_{n/2}$ and their disjunction is implementing

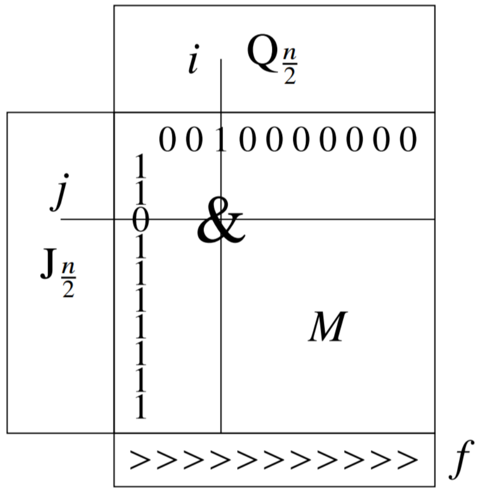

Рис. 1: Schematic diagram of implementation of arbitrary boolean function f from n variables. Here $Q_{\frac{n}{2}}$ is a conjunctive decoder from $\frac{n}{2}$ variables, $J_{\frac{n}{2}}$ is a disjunctive decoder from $\frac{n}{2}$ variables, M is an element matrix, f is implementing function.

the function f. □

Lemma 1. *Any complete basis B in celullar curcuit model consists of not more 641 functional and switching elements.*

Let's define the value $C(B)$ for the arbitrary basis B as a minimum implemention area of the boolean function system of basis B_0' such that inputs and outputs can be placed arbitrary on given sides. Let's denote as $|B|$ the number of all possible elements of the basis B taking into account rotations and reflections. It follows from the lemma 1 that every complete basis B consists of not more than 641 elements and $|B| = 641 * 4 = 2564 = E_{max}$.

Theorem 2 (about the upper estimate in arbitrary basis). *For every boolean function f from n variables in arbitrary complete basis B exists celullar circuit S implementing f and repeating it's inputs one-time with area:*

$$A(S) \leqslant C(B)2^n + O(n2^{\frac{n}{2}}), \quad n \to \infty.$$

Let's denote as $N(n, k)$ the number of cellular circuits with area k implementing different boolean functions from n variables with one output.

Lemma 2 (about the estimate of different circuits number)

$$\log_2(N(n, k)) < \log_2(|B|)k + n\log_2(k + 1) + O(\log_2(k)).$$

Theorem 3 (about the lower estimate). *For the area $A(S)$ of cellular circuit without repeating inputs in the arbitrary basis B is true such an estimate for Shannon function:*

$$A(n) \geqslant \frac{\ln(2)}{\ln(|B|)}2^n.$$

It is true for every $\epsilon > 0$ *and big enough* n

$$A(S) > C2^n(1-\epsilon), C = \frac{\ln(2)}{\ln(|B|)}.$$

Wherein the part of functions f *for which the inequality is not satisfied tends to zero as* n *grows.*

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